Betatron Matching and Dispersion Matching

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Abstract

The TT2-TT10 transfer line optics has been modeled and optimized in order to minimize blow-up at injection into the SPS. Betatron and dispersion matching has been performed for the fixed-target proton and ion beams, as well as for the future LHC beam. Based on the model, tuning knobs were developed to tune independently Twiss parameters, dispersion and dispersion derivative. Coupled to the measurement of the Twiss parameters in the line or in the SPS, they can be used for on-line mismatch correction. The correction mechanisms are discussed and first experimental results are presented.

1 INTRODUCTION

Recently there has been renewed interest in the optics of the TT2-TT10 transfer line between the PS and the SPS machines at CERN. The main reason is that the line will be an important part of the LHC injection chain [1]. Given the tight emittance budget allowed for the LHC beam, Twiss parameters and dispersion must be matched accurately in order to prevent blow-up of the beam at injection into the SPS. Good matching is also of interest for the fixed-target proton and ion beams which use the same beamline: minimizing blow-up at injection results in an increased transmission through the SPS and hence in an increased intensity delivered to the targets. In addition, minimizing beam losses means less activation of machine components.

Proper matching relies both on a consistent model as well as on a precise knowledge of the input parameters at the beginning of the line. Considerable effort was spent on these two items during the 1998 SPS run. In a first step, the complete TT2-TT10 line was modeled using the program package *MAD* [2]. The geometry of the model was cross-checked versus the official CERN survey data [3] and the correct magnetic behaviour of the elements was confirmed in a series of measurements [4]. Then, the initial parameters of all the beams concerned were measured and used as an input for the model.

Based on the measured initial conditions and the verified model, the line was successfully rematched for the LHC beam as well as for the fixed-target proton and ion beams. In the following sections we discuss the matching procedure as well as the results obtained for the various beams. In the last section we present a tuning tool to correct for the unavoidable residual discrepancy between simulation and real machine.

2 LHC BEAM

Extensive studies were carried out during the 1998 SPS run using the 26 GeV/c proton beam which is used for LHC studies. The beam was provided on its own cycle and measurements could be done independently at a minimum interference with the physics program. The relevant beam parameters can be summarized as

momentum:	p	= 26 GeV/c
momentum spread:	dp/p	= 10^{-3}
normalized horizontal emittance:	ε_x	$\approx$ 3.0 μm
normalized vertical emittance:	ε_y	$\approx$ 3.0 μm

Since for the LHC injection a maximum emittance growth of 0.5 μm is allowed from the PS to the SPS[1], accurate Twiss parameter and dispersion matching is mandatory. In particular for this type of beam, where a small beam size coincides with a large momentum spread, blow-up due to dispersion mismatch is a major concern.

2.1 Betatron Matching

The 1998 run was started with an optics matched to initial conditions obtained from a *MAD* simulation of the fast extraction from the PS machine. The geometrical betatron mismatch obtained from a multi-grid measurement in TT10 was about 1.5 in the horizontal and 1.15 in the vertical plane[2]. During the 1998 run, Twiss parameters and dispersion were measured and tracked back to the PS extraction point. With this set of input parameters and the newly generated set of *MAD* files a matching of the complete TT2-TT10 line was performed. The mismatch factor decreased in one iteration to 1.3 in the horizontal and 1.0 in the vertical plane.

2.2 Dispersion Matching

In order to determine the initial values of D and D', the momentum in the PS machine has to be varied. Typically five different settings are applied. The displacement of the beam in both planes is then observed at all monitors in the injection line and in the SPS. Considering the first turn of the SPS as a continuation of the transfer line allows to make use of a large number of monitors the transfer matrices between which are well known. A fit taking into account the

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[1] Note, that only a fraction of this is assigned to mismatch.

[2] These values correspond to the geometrical mismatch factor. The mismatch factor after filamentation is much smaller, see L. Vos and G. Arduini, these proceedings.

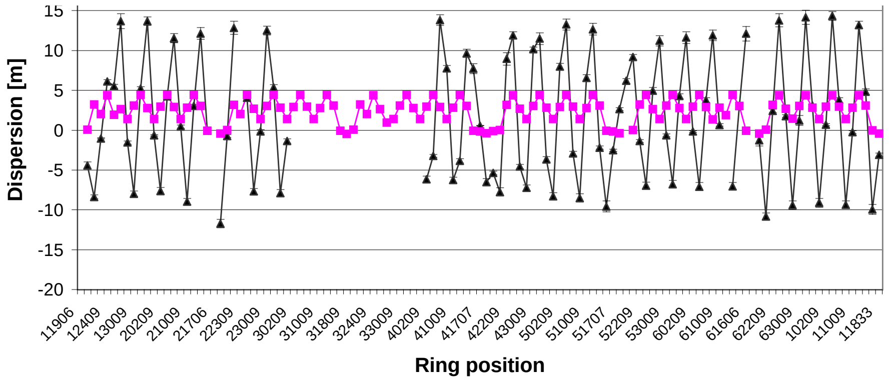

Figure 1: Measured horizontal dispersion of the LHC beam in the SPS before matching the injection line. The triangles represent the measured data, the squares the theoretical values.

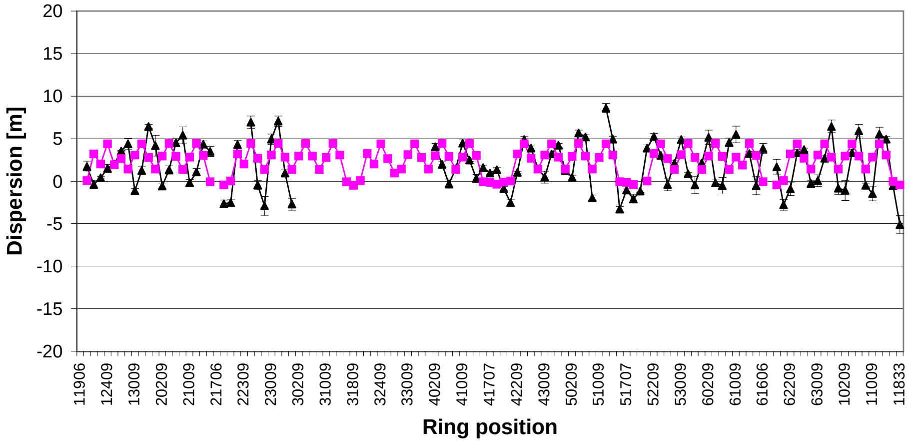

Figure 2: Measured horizontal dispersion of the LHC beam in the SPS after matching the injection line. The triangles represent the measured data, the squares the theoretical values.

measured dispersion at all monitors and the transfer matrices between them yields the initial dispersion at the beginning of the line.

Figure 1 shows the horizontal dispersion function in the SPS before and Fig. 3 after matching the injection line. The dispersion amplitude was brought down to the design values in one iteration. In the vertical plane, the measured dispersion was close to zero which corresponds as well to the theoretical value.

3 FIXED-TARGET PROTON BEAM

Matching the line for the 14 GeV/c fixed-target proton beam[3] turned out to be more complicated than in the case of the LHC beam. The optics for this beam includes an emittance exchange insertion in the TT10 part of the line. The quadrupoles in this section have a fixed setting and cannot be used for matching. Another technical complication is, that the *MAD* model presently used for this beam splits up the line into two parts before and after the emittance exchange. The computation is then done in two steps which makes the matching even more complicated.

The beam parameters for 1998 were as follows:

momentum:	p	= 14 GeV/c
momentum spread:	dp/p	$= 0.5 \cdot 10^{-3}$
normalized horizontal emittance:	ε_x	$\approx$ 10.0 μm
normalized vertical emittance:	ε_y	$\approx$ 7.5 μm

The fixed-target proton beam was provided on the 'main cycle' of the SPS. In order not to interfere with the physics program, measurements could only be done during dedicated MD time.

3.1 Betatron Matching

Running the line with the 1997 optics, the mismatch factor was determined from a multi-grid measurement in TT10 to

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[3] This beam is frequently referred to as 'continuous transfer' or 'ct' beam.

about 2.0 in both planes. The same effort as in the case of the LHC beam was made to determine a set of initial beam parameters to be used as input for a rematching of the line. The Twiss parameters were determined using the SEM monitors in TT2 and TT10 and tracked back to the beginning of the line. Based on the measured initial Twiss parameters, the line was rematched. Several iterations were necessary to achieve a reasonable result. For the best optics, we found a horizontal mismatch of 1.3 and a vertical mismatch of 1.1.

3.2 Dispersion Matching

The dispersion along the line and the SPS first turn was measured and the initial conditions determined as in the case of the LHC beam. The horizontal dispersion was already before rematching close to the theoretical values. It did therefore not improve any more. The vertical dispersion, which was of the order of 3 m before changing the optics, decreased to almost zero after dispersion matching.

4 FIXED-TARGET LEAD ION BEAM

The optics model used for the fixed-target lead ion beam corresponds exactly to the one used for the LHC beam, that is the optics without emittance exchange. The beam parameters of the lead ion beam can be summarized as follows:

momentum (per nucleon):	p	= 5.11 GeV/c
momentum spread:	dp/p	$= 1.5 \cdot 10^{-4}$
normalized horizontal emittance:	ε_x	$\approx$ 3.4 μm
normalized vertical emittance:	ε_y	$\approx$ 1.9 μm

4.1 Betatron Matching

The 1998 run was started with the optics used during the last lead ion run in 1996. The mismatch factor was determined to about 1.8 - 1.9 in the horizontal and 2.1 in the vertical plane. Betatron matching was performed as for the other beams, and for the matched optics a mismatch factor of 1.2 in the horizontal and 1.3 in the vertical plane was found.

Loading the matched optics resulted immediately in an increase of transmission through the SPS of about 20%. Figure 3 shows the average number of ions per cycle during the day when the optics was changed. A step can clearly be seen at the time when the optics was put into operation.

4.2 Dispersion Matching

Since the momentum spread of the beam was extremely small, dispersion mismatch was not a major concern. However, it was decided to measure the dispersion and to perform a full Twiss parameter and dispersion matching. Before matching, the horizontal dispersion had values of up to 15 m to be compared with a maximum theoretical value of 5 m. It decreased to a maximum amplitude of 10 m after rematching, which is still about twice the design value. In the vertical plane, a significant decrease of the dispersion amplitude from about 6 m to about 1 m was found. This is to be compared with the theoretical value of zero vertical dispersion in the SPS.

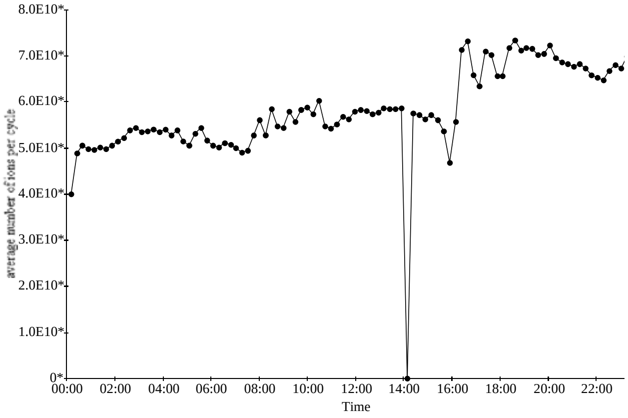

Figure 3: Average number of Pb ions per cycle logged over 24 hours. The step function in efficiency at 16:25 is due to the installation of a new, matched optics in TT10. The drop at 14:00 is due to a power supply failure.

5 TUNING TOOL FOR ON-LINE CORRECTION

In the previous sections we have shown the results of global betatron and dispersion matching. Although accurate matching was performed in the simulation, some residual mismatch is found in all cases. This is both due to an unavoidable discrepancy between model and real machine and due to errors in the measurement of the initial beam parameters.

To correct for these effects, tuning knobs were developed to tune independently the beam parameters. They are based on the inversion of the matrix $\left(\frac{\partial \Delta_i}{\partial K_j}\right)$ $(i, j = 1, ...8)$ where $\Delta_i = (\alpha_{h(v)}, \beta_{h(v)}, D_{h(v)}, D'_{h(v)})$ and K_j is the strength of the j^{th} matching quadrupole. In a first attempt, only the eight independent quadrupoles of the TT10 matching section have been considered. The 64 coefficients of the matrix were obtained from the *MAD* simulation. Inverting the matrix yields immediately the change of quadrupole strength that has to be applied in order to obtain a given change of any of the beam parameters.

Frequently, as in our case, the coefficient matrix for a given optics will be either singular or numerically close to singular. In these cases, it cannot easily be inverted. Singular value decomposition (*SVD*) algorithms provide a tool to diagnose a matrix and to solve systems of equations of the given form even for ill-conditioned matrices [6]. An *SVD* algorithm was applied to recondition the coefficient matrix

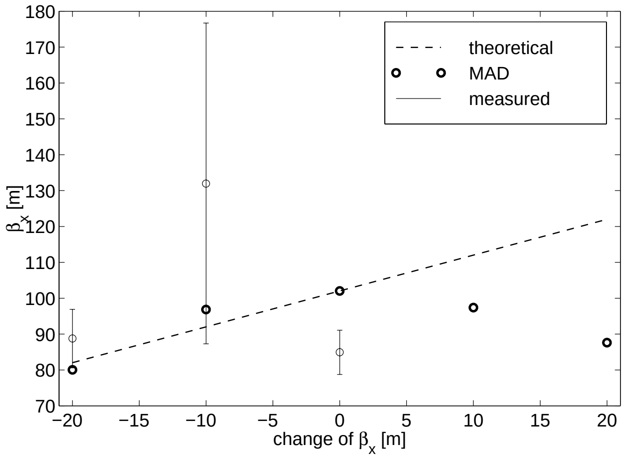

Figure 4: Horizontal β-function at the TT10 injection point versus variation applied using the tuning knob. Starting from a matched optics ($\Delta\beta$ = 0), the β-function is decreased by 10 m and 20 m. The expected, simulated and measured behaviour of the β-function is shown.

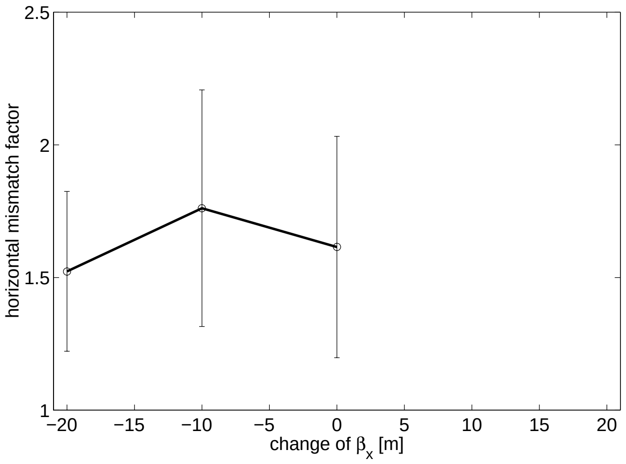

Figure 5: Measured horizontal mismatch factor versus variation applied using the tuning knob. Starting from a matched optics ($\Delta\beta$ = 0), the β-function is decreased by 10 m and 20 m.

and the resulting tuning tool was tested with the model. The simulation showed that the knob worked very well for all beam parameters except the horizontal β-function.

5.1 *Test with Beam*

The tuning tool was tested with beam, using the SPS mismatch monitor [7]. This system is based on a turn-by-turn measurement of the beamsize with an OTR screen and a fast CCD camera. The oscillation of the beamsize indicates betatron mismatch at injection into the SPS. Preliminary tests by drastically detuning the TT10 optics had proved the sensitivity of the monitor. While the oscillation of the beamsize gives only a qualitative measure of the mismatch, it is important to obtain also the values of α, β and the mismatch factor. This can be done by measuring the beam profile at three successive turns in the SPS. The Twiss parameters can then be obtained in the same way as from a multi-grid measurement in a transfer line.

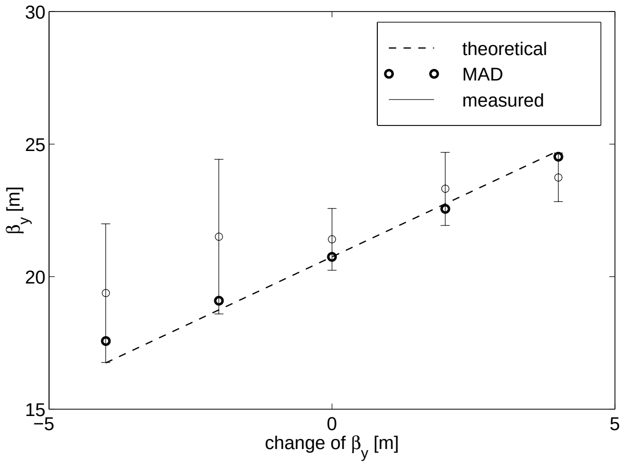

Figure 6: Vertical β-function at the TT10 injection point versus variation applied using the tuning knob. Starting from a matched optics ($\Delta\beta$ = 0), the β-function is increased and decreased by 2 m and 4 m. The expected, simulated and measured behaviour of the β-function is shown.

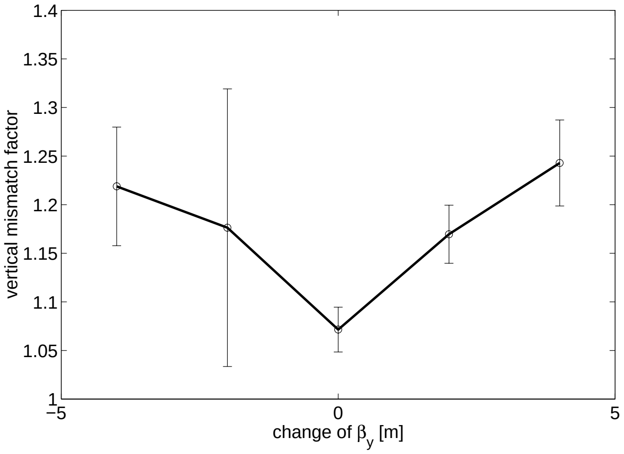

Figure 7: Measured vertical mismatch factor versus variation applied using the tuning knob. Starting from a matched optics ($\Delta\beta$ = 0), the β-function is increased and decreased by 2 m and 4 m.

Since measurement of the Twiss parameters can be performed quickly, while measurement of the dispersion is time consuming, it was decided to detune the horizontal

beam	β mismatch (geometrical)	β mismatch (filamentation)	dispersion mismatch (geometrical)	dispersion mismatch (filamentation)
LHC	1.3 (h) / 1.0 (v)	1.03 (h) / 1.00 (v)	4.63 (h) / 1.18 (v)	1.66 (h) / 1.00 (v)
ct	1.3 (h) / 1.1 (v)	1.03 (h) / 1.00 (v)	1.46 (h) / 1.59 (v)	1.02 (h) / 1.03 (v)
Pb	1.2 (h) / 1.3 (v)	1.02 (h) / 1.03 (v)	1.27 (h) / 1.04 (v)	1.01 (h) / 1.00 (v)

Table 1: Measured betatron and dispersion mismatch factors for the various beams. The relevant quantity is the mismatch factor after filamentation.

and vertical β-funcions starting from a matched setting and to measure Twiss parameters and mismatch factor using the SPS mismatch monitor. The β-funcions were detuned by ±10% and ±20%. Figure 4 shows the result for the horizontal plane. The plot shows the expected change of the β-function (dashed line), the result obtained from the simulation and the measured values. It can be seen, that already in the simulation the expected variation is not achieved. As far as the measurement is concerned, only the points for $\Delta\beta$ = 0, -10 m and -20 m could be measured due to a technical problem. For the initial setting ($\Delta\beta$ = 0), the measured value lies already below the theoretical one. This means that this setting is not perfectly matched, which is consistent with a measured mismatch factor of 1.3 - 1.4. In general, the fluctuation is very large due to a horizontally unstable beam which makes it impossible to draw a conclusion.

From the measured Twiss parameters, the mismatch factor can be computed. Figure 5 shows the horizontal mismatch factor versus change of β_x for the same measurement. The large fluctuations in the horizontal plane make a conclusion impossible.

The same measurement was done in the vertical plane. The beam was much more stable and all five settings could be measured. Figure 6 shows theoretical, simulated and measured values of β_y for five different settings of the tuning knob. All data agree within the statistical error.

From the same measurement, the vertical mismatch factor was computed. The result is shown in Fig. 7. For the matched optics ($\Delta\beta$ = 0), a vertical mismatch factor of 1.1 is found which is in perfect agreement with the result obtained from a multi-grid measurement in TT10. Detuning the β-function at the injection point in both directions leads to an increase of the mismatch factor as expected.

6 CONCLUSION

Accurate measurement of the beam parameters in the TT2-TT10 injection line and consequent matching of the optics to the measured values has significantly improved the SPS performance during the 1998 run.

For the LHC beam, the geometrical mismatch factor decreased in one iteration from 1.5 (h) and 1.15 (v) to 1.3 (h) and 1.0 (v). The dispersion with the matched optics is now in both planes close to the theoretical values.

For the fixed-target proton beam, the mismatch factor could be decreased from 2.0 in both planes to 1.3 (h) and 1.1 (v). The horizontal dispersion was already close to the theoretical values and did not improve any more. The vertical dispersion, which had a maximum amplitude of 3 m before matching, is now close to zero (theoretical value).

A spectacular success was achieved for the fixed-target lead ion beam. By accurate Twiss parameter matching, the transmission through the SPS and hence the intensity delivered to the targets could be increased in one step by about 20%. The measured mismatch factors were 1.9 (h) / 2.1 (v) before and 1.2 (h) / 1.3 (v) after matching. The dispersion could slightly be improved in the horizontal plane and significantly improved in the vertical plane.

On top of the matched optics, a tuning tool was developed and successfully tested for the LHC beam.

Table 1 shows the measured mismatch factors for betatron and dispersion mismatch for the various beams. While the measured values correspond to the geometrical mismatch factor, the mismatch after filamentation is much smaller [5]. A critical value is the blow-up due to dispersion mismatch for the LHC beam. This problem is likely to be overcome, either by another iteration or by applying the tuning tool which has been shown to work excellent for D and D' in the simulation.

7 ACKNOWLEDGEMENTS

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