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Emittance Measurements For Future LHC Beams Using The PS Booster Measurement Line

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Abstract

The CERN PS Booster measurement line contains three pairs of SEM grids separated by drift space that measures the beam size in both planes. The combined analysis of these grids allows calculating a value for the transverse beam emittances. The precision of such a measurement depends on the ratio of RMS beam size and wire spacing. Within the LIU-PSB upgrade the extraction kinetic energy of the PSB will be increased from the current 1.4 GeV to 2.0 GeV. This will result in smaller transverse beam sizes for some of the future beams. The present layout of the transverse emittance measurement line is reviewed to verify if it will satisfy future requirements.

Keywords: BTM, emittance measurement, LIU, PS Booster, SEM grids.

Contents

1 The BTM line

The scheme of the PS Booster (PSB) recombination lines is shown in Fig. 1. In this paper we study the line marked in red, the Booster Transfer Measurement line (BTM). This line has two main purposes. The first one is to transfer the beam to the dump, the second one is to perform measurements of the transverse emittance. The latter is done with the three pairs of Secondary-Electron Emission Monitor grids (SEM). They measure the beam profile for each plane and provide the 1- beam size at the location of each grid. The combined measurement of the three grids gives the emittance in the respective plane.

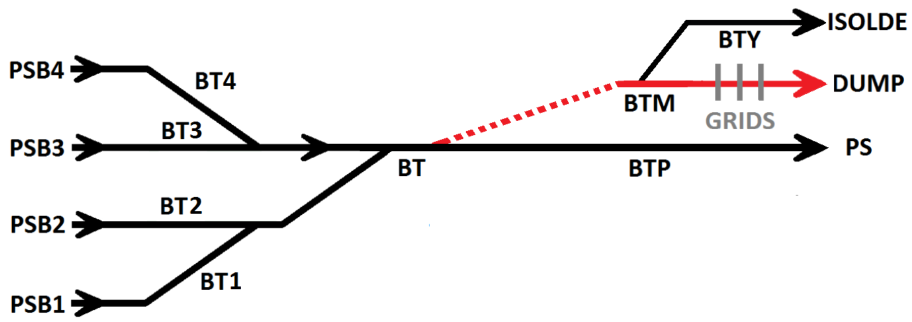

Fig. 1: Scheme of the PSB recombination lines. The BTM line is marked in red.

There are different optics configurations optimized for the vertical and for the horizontal plane [1]. Each configuration fulfills the following relations on the Twiss functions for the respective plane [2]:

$$\alpha_2 = 0; \tag{1}$$

$$\Delta\mu_{1,2} = \Delta\mu_{2,3} = \pi/3; \tag{2}$$

$$min\left[\frac{D_i^2}{\beta_i} + \left(\alpha_i\frac{D_i}{\sqrt{\beta_i}} + \sqrt{\beta_i}D_i'\right)^2\right]. \tag{3}$$

Here the subindex refers to the grid (2 for the central grid and 1,3 for outer ones). Equation 1 gives a symmetry of the Twiss functions in grids 1 and 3. The complete full phase space coverage is ensured by Eq. 2. Finally, Eq. 3 aims at minimizing the error coming from the dispersion, as this value is proportional to the module of the normalized dispersion vector.

Figure 2 shows a screenshot of the control room application of the PSB. A Gaussian function is fitted to the beam profile and the combined measurement gives the emittance.

2 New optics for the BTM line

A new optics was computed [2] in order to relax some specifications of the new hardware required for the 2.0 GeV upgrade within the LIU program [3]. In particular, this new optics aims at reducing the beam size at one of the bending magnets, and allows to reduce its gap height, easying its design. The new optics will also improve the emittance measurements in the BTM line, as the conditions in Eqs. 1, 2, 3 are met with better precision. However, at the energy of 2.0 GeV, the beam sizes at the location of the grids become smaller by approximately 10%. Therefore it is necessary to re-evaluate the suitability of the current measurement hardware configuration for all future beams.

3 Future Measurements

First of all, we have to consider the wire spacing of the present grids. For the inner couple of SEM grids (grid 2), the current separation between wires is 0.5 mm. The outer grids (1,3) can afford larger wire separation, 1 mm, as the beam size is expected to be larger at their location. Table 1 summarizes the characteristics of the most limiting future beams to be measured in the BTM line [4]. LHC and BCMS beams are intended for LHC physics purposes, while the probe beam is used for setting up and tests.

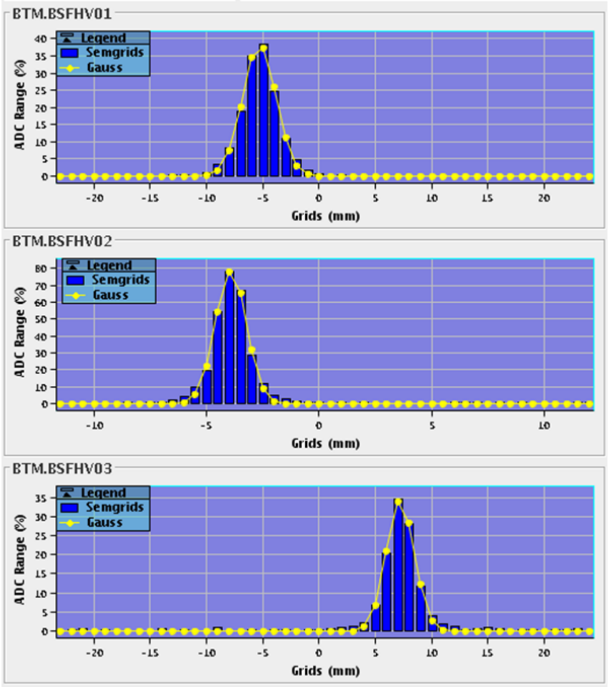

Fig. 2: Capture of the application in the control room, showing the beam profile for the three grids and their respective fit to a Gaussian function.

Table 1: Beam types at 2.0 GeV, transverse normalized emittance (ϵ_N) and RMS momentum spread (σ_δ).

Beam	$\epsilon_N[\mu m]$	$\sigma_\delta[\times 10^{-3}]$
LHC	1.63	1.5
BCMS	1.19	1.1
probe	1.0	1.07

The expected beam sizes in the horizontal plane at the location of the SEM grids are shown in Tab. 2. There are no differences between the present and the future optics in terms of beam size at the grids. With regard to the vertical plane, the $1-\sigma$ values are shown in Tab. 3.

Table 2: Beam dimensions on the grids for the horizontal measurement. The same value applies for both current and proposed optics.

Beam	LHC	BCMS	Probe
$\sigma_{x,1}[mm]$	2.8	2.2	2.1
$\sigma_{x,2}[mm]$	0.9	0.8	0.7
$\sigma_{x,3}[mm]$	2.8	2.2	2.1

Table 3: Beam dimensions on the grids for the vertical measurement (current and proposed optics).

	Beam	LHC	BCMS	Probe
Current optics	$\sigma_{y,1}[mm]$	1.7	1.5	1.4
	$\sigma_{y,2}[mm]$	0.9	0.7	0.7
	$\sigma_{y,3}[mm]$	1.9	1.6	1.5
Proposed optics	$\sigma_{y,1}[mm]$	1.8	1.5	1.4
	$\sigma_{y,2}[mm]$	0.9	0.8	0.7
	$\sigma_{y,3}[mm]$	1.8	1.5	1.4

For the worst case, the beam sizes are 40% larger than the wire separation.

4 Influence of Beam Size Precision on Emittance Measurement

The errors in the emittance measurement can be deduced for the general case of three grids separated by a distance L and with a symmetric optics with respect to the central grid, that is, $\alpha_2 = 0$. This is the case for the BTM line as sketched in Fig. 3.

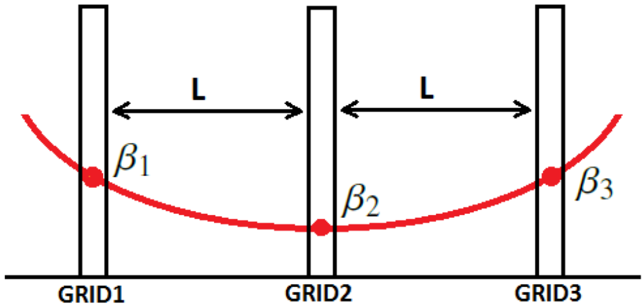

Fig. 3: Schematic of a three-grid emittance measurement section, like the one of the BTM line.

The emittance and the Twiss functions on the first grid are obtained from the betatron beam size on each monitor, $\sigma_i = \sqrt{\epsilon \beta_i}$ [5]. By applying these relations to the case on Fig. 3, we get:

$$\begin{pmatrix} \beta_1 \epsilon \\ \alpha_1 \epsilon \\ \gamma_1 \epsilon \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3/4L & -1/L & 1/4L \\ 1/2L^2 & -1/L^2 & 1/2L^2 \end{pmatrix} \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \end{pmatrix}. \tag{4}$$

The emittance is then

$$\epsilon = \sqrt{AC - B^2}; \tag{5}$$

where

$$A = \sigma_1^2, \tag{6}$$

$$B = \frac{3}{4L}\sigma_1^2 - \frac{1}{L}\sigma_2^2 + \frac{1}{4L}\sigma_3^2, \tag{7}$$

$$C = \frac{1}{2L^2}\sigma_1^2 - \frac{1}{L^2}\sigma_2^2 + \frac{1}{2L^2}\sigma_3^2. \tag{8}$$

The error in emittance is

$$\Delta\epsilon = \frac{1}{2\epsilon}\sqrt{A^2\Delta C^2 + C^2\Delta A^2 + 4B^2\Delta B^2}; \tag{9}$$

where

$$\Delta A^2 = 4\sigma_1^2\Delta\sigma_1^2, \tag{10}$$

$$\Delta B^2 = \frac{9}{4L^2}\sigma_1^2\Delta\sigma_1^2 + \frac{4}{L^2}\sigma_2^2\Delta\sigma_2^2 + \frac{1}{4L^2}\sigma_3^2\Delta\sigma_3^2, \tag{11}$$

$$\Delta C^2 = \frac{1}{L^4}\sigma_1^2\Delta\sigma_1^2 + \frac{4}{L^4}\sigma_2^2\Delta\sigma_2^2 + \frac{1}{L^4}\sigma_3^2\Delta\sigma_3^2. \tag{12}$$

As the optics is symmetric with respect to grid 2 (Fig. 3),

$$\sigma_1 = \sigma_3 = \sqrt{1 + \left(\frac{L}{\beta_2}\right)^2}\,\sigma_2. \tag{13}$$

It is now assumed that the betatron error is the same in the outer grids,

$$\Delta\sigma_1 = \Delta\sigma_3. \tag{14}$$

It has been taken into account that the beam sizes measured by the grids ($\sigma_{i,m}$) are larger than the betatron contribution, as there is a dispersive term. They are related as:

$$\sigma_i^2 = \sigma_{i,m}^2 - D_i^2\sigma_\delta^2, \tag{15}$$

where D_i is the dispersion at the corresponding grid and σ_δ the RMS momentum spread. We also assume that $D_2 = 0$ which implies that $D_1 = -D_3$, then $\sigma_2 = \sigma_{2,m}$. By defining the factor f as

$$f = \frac{\sigma_{1,m}}{\sigma_1} = \frac{\sigma_{3,m}}{\sigma_3}. \tag{16}$$

From Eqs. 14, 15, 16, the errors are related as

$$\Delta\sigma_1 = \Delta\sigma_3 = f\Delta\sigma_{1,m} = f\Delta\sigma_{3,m}; \tag{17}$$

and for the central grid

$$\Delta\sigma_2 = \Delta\sigma_{2,m}. \tag{18}$$

By substituting Eq. 10, Eq. 11, Eq. 12 into Eq. 9 and introducing Eq. 13, Eq. 14, Eq. 16 and Eq. 17, we get

$$\frac{\Delta\epsilon}{\epsilon} = \sqrt{C_1 f^2 \left(\frac{\Delta\sigma_{1,m}}{\sigma_{1,m}}\right)^2 + C_2 \left(\frac{\Delta\sigma_{2,m}}{\sigma_{2,m}}\right)^2}; \tag{19}$$

where

$$C_1 = \frac{\beta_2^4}{4L^4}\left[16\left(1+\frac{L^2}{\beta_2^2}\right)^4 - 28\left(1+\frac{L^2}{\beta_2^2}\right)^3 + 14\left(1+\frac{L^2}{\beta_2^2}\right)^2\right], \tag{20}$$

$$C_2 = \frac{\beta_2^4}{4L^4}\left[20\left(1+\frac{L^2}{\beta_2^2}\right)^2 - 32\left(1+\frac{L^2}{\beta_2^2}\right) + 16\right]; \tag{21}$$

which indicates a dependence on the ratio L/β_2. On the other hand, this ratio is fixed by Eq. 2:

$$\frac{L}{\beta_2} = \tan(\frac{\pi}{3}). \tag{22}$$

5 Beam Size Precision and Grid Separation

The error in the measured beam size, $\Delta\sigma_m$, is related to the number of wires of the SEM grid. The main source of error for the individual wires is electronic noise. A study was made to determine the relation between density of wires per beam size unit (M_i) and the precision on the beam size [6]. Numerical simulations were done for different beam profiles and different SEM grid configurations. It was found that the error in the measured beam size and the density of wires is roughly described by the following relation:

$$\frac{\Delta\sigma_{i,m}}{\sigma_{i,m}} = \frac{1}{(5M_i)^2} = \frac{1}{5^2}\left(\frac{\Delta W_i}{\sigma_{i,m}}\right)^2. \tag{23}$$

Then, from Eq. 16, Eq. 19, Eq. 20, Eq. 21 and Eq. 23 we can get the relative emittance error as a function of the wire separation in the grids ($\Delta W_1 = \Delta W_3$):

$$\frac{\Delta\epsilon}{\epsilon} = \frac{2}{75}\sqrt{158\left(1+\frac{D_1^2\sigma_\delta^2}{\sigma_1^2}\right)\left(\frac{\Delta W_1}{\sigma_{1,m}}\right)^4 + 13\left(\frac{\Delta W_2}{\sigma_2}\right)^4}; \tag{24}$$

First of all, we note that the wire density in the outer grids has a much larger contribution to the emittance error than the wire density of the central one. The relative emittance error in the horizontal plane (from Eq. 24) is shown in Fig. 4 for the particular case $D_1 = -1.4$ m, $\sigma_2 = 0.7$ mm, $\sigma_{1,m} = 2.2$ mm, $\sigma_1 = 1.5$ mm (BCMS beam, Table 2). The present SEM grid configuration, with $W_1 = 1$ mm and $W_2 = 0.5$ mm yields an error of 11%.

With regard to the vertical plane, the error in the emittance measurement is shown in Fig. 5. The case corresponds to $D_1 = 0$, $\sigma_2 = 0.7$ mm, $\sigma_{1,m} = \sigma_1 = 1.5$ mm (BCMS beam).

We can appreciate that the error is larger than for the horizontal case. The reason is that the beam is smaller in the outer grids with respect to the horizontal case, due to the fact that vertical dispersion is zero.

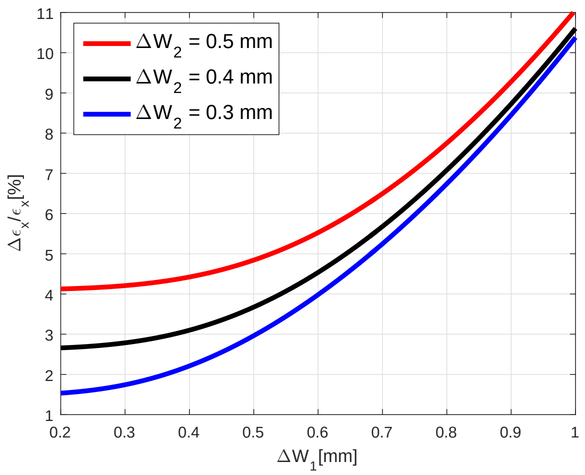

Fig. 4: Horizontal emittance relative error as a function of the wire separation in grids 1 and 3, for different values of the wire separation in grid 2.

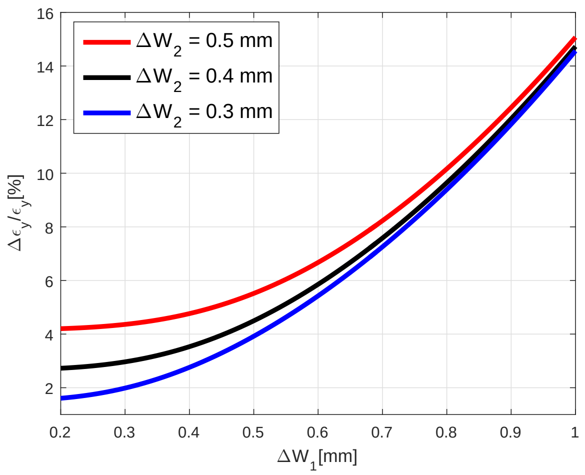

Fig. 5: Vertical emittance relative error as a function of the wire separation in grids 1 and 3, for different values of the wire separation in grid 2.

In order to achieve an error less than 5% in the horizontal plane, one could keep the present wire separation of grid 2 but decrease the outer distance for the outer grids as well to 0.5 mm. This would yield an error of $\sim 5.5\%$ for the vertical emittance.

For all these calculations, we have assumed no errors in the optics functions and on the momentum spread. The plots in Figs. 4 and 5 should be then seen as a limit case on the minimum emittance error.

6 Alternatives

The first reasonable option to increase the precision would be to tilt the grids so that the effective projected wire spacing would be reduced. However, it should be evaluated if this will allow to measure the large emittance beams, as the size of the monitor in number of σ would be reduced too. Another option would be to reduce the wire spacing, but this would mean to build new equipment. One could also use the same grids and displaced them in the transverse direction of the measurement for each shot. Like that, some precision would be gained in the beam profile reconstruction. The problem is that the measurement would be masked by shot-to-shot variations, as it will no longer be a single-shot measurement as today. This is the same problem of one would install a wire scanner in the line; we would need even more shots to have a full measurement of the beam profile. The use of OTR (Optical Transition Radiation) screens is also discarded due to the low energy of the beams.

7 Conclusion

We have analyzed the present BTM emittance measurement system, consisting of three pairs of grids. The precision of the system is dependent on the beam size, which will shrink at the upgraded kinetic energy of 2.0 GeV. In order to analyze the influence of the beam size on the emittance error, we have derived a general relation, which is valid for any configuration of three grids with a symmetric optics with respect to the central one and a phase advance of $\pi/3$ between grids. By combining this result with the relation of the precision on beam size measurement with the present wire separation in the grids, we can conclude that the system should be upgraded to reach a precision of 5%. In this case, a system where the three grids have a wire separation of 0.5 mm would be needed.

8 References

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